# 第26回微分方程及相關領域年會

**26th Annual Meeting on Differential Equations and Related Topics** 



臺灣大學數學系

#### 微分方程研討會

在林長壽院士提議下第一屆方程年會於 1993 年在中正大學舉辦,爾後方程 年會成為方程領域學者每年固定的學術活動,可以說是國內次領域中歷史最悠久 的年會。這個會議提供了國內從事方程相關領域研究學者或學生,交流彼此研究 成果及心得重要的平台。本屆方程年會將由台灣大學數學系主辦,歡迎各位學術 先進,蒞臨指導。

#### 歷年舉辦地點

第1屆 1993 中正大學 第2屆 1994 交通大學 第3屆 1994.12 中央研究院(暨中日算子理論聯合研討會) 第4屆 1996 清華大學 第5屆 1997 中央大學 第6屆 1998 師範大學 第7屆 1999 中興大學 第8屆 2000 中山大學 第9屆 2001 成功大學 第10 届 2002 彰化師範大學 第11 屆 2002.12 靜宜大學 第12 屆 2004 清華大學(暨分析研討會) 第13 屆 2005 中正大學 第14 届 2006 中央研究院(暨分析研討會) 第15 屆 2006.12 南台科技大學 第16 屆 2008 交通大學 第17屆 2009 高雄大學 第18 屆 2010 台灣大學 第19屆 2011 成功大學 第20 届 2012 淡江大學 第21 屆 2013 中央大學 第22 屆 2014 清華大學 第23 屆 2015 臺東大學 第24 屆 2016 中山大學 第25 屆 2017 交通大學 第26 届 2018 台灣大學

#### 籌備委員(依姓氏筆畫排序)

王振男、林太家、夏俊雄、陳俊全

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10:50-11:20	Chia-Chieh Jay Chu	10:50-11:20	Hsin-Yuan Huang
11:30-12:00	Ming-Cheng Shiue	11:30-12:00	Ting-Jung Kuo
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MS2 (Semilinear Elliptic Equations with Applications)

MS3 (Reaction-Diffusion Equations and Related Topi

MS4 (Kinetic Theory and Gas Dynamics)

年會會場(Venue):

MS6 (Mathematical Biology)

臺灣大學天文數學館101、102室 MS7 (Dynamical Systems and Differential Equations) Room 101 & Room 202, Astronomy-

MS8 (Inverse Problems and Related Questions)

MS5 (Fluid equations and Schrodinger equations)

Mathematics Building, NTU

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10:50-11:20	Chia-Chieh Jay Chu Volumetric Variational Problems for Partial Differential Equations on Manifolds	10:50-11:20	Hsin-Yuan Huang Bubbling Solutions for the Liouville System		
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12:00-12:30	<b>Wei-Fan Hu</b> Spontaneous autophoretic motion of colloidal particles in two-dimensional space	12:00-12:30	Wen Yang On the uniqueness of the steady state in a Keller- Segel model		
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## Some recent developments on a singular predator-prey model

#### Jong-Shenq Guo

#### Tamkang University E-mail:jsguo@mail.tku.edu.tw

#### Abstract

We consider a singular predator-prey model which describes the interaction between native birds and introduced cats in an island. Taking into account the spatial dependence (or, random movements), the dynamics becomes much more involved than the corresponding kinetic system. We shall describe some recent developments of this reaction-diffusion system for different values of growth rates for birds and cats. There is the so-called spatio-temporal oscillations, namely, solutions asymptotically become spatial-homogeneous and time-periodic. On the other hand, we analyze the corresponding shadow system when the diffusion coefficient of birds becomes very large. Some global and non-global existence results for this shadow system are obtained. Finally, some open problems are to be given. This talk is based on joint works with Arnaud Ducrot and Masahiko Shimojo.

## Partial differential equations related to fluids with partial or fractional dissipation

#### Jiahong Wu

#### Department of Mathematics Oklahoma State University E-mail:**jiahong.wu@okstate.edu**

#### Abstract

There have been substantial recent developments on several partial differential equations from fluid dynamics with partial or fractional dissipation. This talk summarizes results on the global existence and regularity problem for the 3D Navier-Stokes equations with partial hyperdissipation, the surface quasi-geostrophic equation, the 2D Boussinesq equations and the 2D magnetohydrodynamic equations with partial or fractional dissipation.

## Inverse problems for viscoelasticity systems by Carleman estimates

#### Masahiro YAMAMOTO

#### Graduate School of Mathematical Sciences, The University of Tokyo E-mail:**myama@ms.u-tokyo.ac.jp**

#### Abstract

We consider several coupling systems including the linear viscoelasticity equation and the Kelvin-Voigt model. The principal parts are not only coupled but also these systems are characterized by hyperbolic-integral isotropic Lame equations. By Carleman estimates, we discuss the uniqueness and the stability for inverse problems of determining spatially varying coefficients and/or factor in sources.

The coupling with integration terms yields difficulties in establishing relevant Carleman estimates and there are few results concerning the inverse problems although these systems are important physically.

We establish Carleman estimates with partial boundary data and the Lipschitz stability for those inverse problems.

This is a joint work with Oleg Y. Imanuvilov (Colorado State University, USA).

## Strong solutions of the Navier-Stokes equations based on the maximal Lorentz regularity theorem in Besov spaces

#### Hideo KOZONO & Senjo SHIMIZU

Waseda University & Kyoto University E-mail:kozono@waseda.jp & shimizu.senjo.5s@kyoto-u.ac.jp

#### Abstract

We show existence and uniqueness theorem of local strong solutions to the Navier-Stokes equations with arbitrary initial data and external forces in the homogeneous Besov space with both negative and positive differential orders which is an invariant space under the change of scaling. If the initial data and external forces are small, then the local solutions can be extended globally in time. Our solutions also belong to the Serrin class in the usual Lebesgue space. As an application, in the 3D case we can handle such singular data for both initial vorticity and external force with their support located on the sphere. The method is based on the maximal Lorentz regularity theorem of the Stokes equations in the homogeneous Besov spaces.

### Extremal spectral gaps for periodic Schrödinger operators

### Chiu-Yen Kao<sup>\*</sup> and Braxton Osting

#### \*Department of Mathematical Sciences, Claremont McKenna College, Claremont, CA E-mail:ckao@cmc.edu

Department of Mathematics, University of Utah, Salt Lake City, UT E-mail:osting@math.utah.edu

#### Abstract

The spectrum of the Schrödinger operator with periodic potential generally consists of bands and gaps. In this talk, for fixed m, we consider the problem of maximizing the gap-tomidgap ratio for the m-th spectral gap over the class of potentials which have fixed periodicity and are pointwise bounded above and below. We prove that the potential maximizing the m-th gap-to-midgap ratio exists. In one dimension, we prove that the optimal potential attains the pointwise bounds almost everywhere in the domain and is a step-function attaining the imposed minimum and maximum values on exactly m intervals. Optimal potentials are computed numerically using a rearrangement algorithm and found to be periodic. In two-dimensions, we develop an efficient rearrangement method for this problem based on a semi-definite formulation and apply it to study properties of extremal potentials. We show that, provided a topological assumption about maximizers holds, a lattice of disks maximizes the first gap-tomidgap ratio in the infinite contrast limit. Using an explicit parametrization of two-dimensional Bravais lattices, we also consider how the optimal value varies over all equal-volume Bravais lattices.

## Volumetric variational problems for partial differential equations on manifolds

#### Jay Chu

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#### Abstract

Partial differential equations on manifolds are important for many areas, such as materials science, fluid dynamics and biology. The computation of such problems can be costly and difficult when the manifolds have complicated structures. Traditional approaches require discretization on manifolds and projecting derivatives onto the tangent spaces of the manifolds. We introduce volumetric variational problems for solving such PDE's. We start with variational problems on manifolds and change them into extended problems in Eulerian formulation. The extended PDE's can be solved by many sophisticated numerical methods, such as finite element or finite difference methods. Based on special properties of the solutions, we design a special treatment for boundary conditions. By Fourier and Laplace transformation, we analysis the method and show that it is stable for elliptic and parabolic type of equations. However, it is not numerically stable for hyperbolic type equation. The instability can be fixed by modifying equations or reinitialization. Some numerical experiments are presented in the talk. This is a joint work with Richard Tsai.

## Convergence and Stability of the MAC scheme for Stokes/Darcy coupling problems based on finite difference methods

#### Ming-Cheng Shiue

#### Department of Appplied Mathematics, National Chiao Tung University E-mail:mingcheng.shiue@gmail.com

#### Abstract

In this talk, to begin with, Stokes/Darcy coupling flows which arise from physical models such as biology, engineering and geophysical fluid dynamics are considered. In the literature, there are many numerical methods related to finite element methods applied to solve this coupling problem. Unlike finite element methods, due to that there is lack of natural variational formulation, in general, the analysis of the scheme based on finite difference methods becomes complicate. In this work, the MAC scheme for this coupling problem based on finite difference methods is used. Convergence and stability of the scheme will be presented. The second part will present the development of numerical schemes for Navier-Stokes and Darcy coupling problems based on projection methods. Numerical simulations demonstrate the results which match the case of only Navier-Stokes equations that also are computed using the projection method. These are joint research works with Ming-Chih Lai and Kian Chuan Ong.

## Spontaneous autophoretic motion of colloidal particles in two-dimensional space

#### Wei-Fan Hu

#### E-mail:wfhu@nchu.edu.tw

#### Abstract

It is well-known that an asymmetrically patterned colloidal particle (Janus particle) interacting chemically with a surrounding solute demonstrates a self-propelled motion. Such particle is assumed to be suspended in a Stokes fluid with force- and torque-free constraints on its surface. We numerically demonstrate that the self-generated solute gradients is unnecessary for locomotion and a spontaneous autophoretic motion can be achieved for an isotropic particle when the Péclet number is above a critical number.

## Existence and multiplicity of nontrivial solutions for a biharmonic equation

Jifeng Chu, Juntao Sun and Tsung-fang Wu

#### Abstract

In this talk, we will study a class of nonlinear biharmonic equations with p-Laplacian

$$\left\{ \begin{array}{ll} \Delta^2 u - \beta \Delta_p u + \lambda V\left(x\right) u = f\left(x, u\right) & \text{ in } \mathbb{R}^N, \\ u \in H^2(\mathbb{R}^N), \end{array} \right.$$

where  $N \ge 1$ ,  $\beta \in \mathbb{R}$ ,  $\lambda > 0$  is a parameter and  $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$  with  $p \ge 2$ . Unlike most other papers on this problem, we replace Laplacian with *p*-Laplacian and allow  $\beta$  to be negative. Under some suitable assumptions on V(x) and f(x, u), we obtain the existence and multiplicity of nontrivial solutions for  $\lambda$  large enough. The proof is based on variational methods as well as Gagliardo-Nirenberg inequality.

### Bubbling solutions for the Liouville system

#### Hsin-Yuan Huang

#### Department of Applied Mathematics, National Chiao-Tung University E-mail:hyhuang@math.nctu.edu.tw

#### Abstract

In this talk, I will briefly introduce the recent developments on the Liouville system. The system is related to several models of Chemistry, Ecology and Physics. My recent existence result on the bubbling solutions will be present.

## A connection of generalized Lamé equation and the mean field equation

Kuo, Ting-Jung

#### National Taiwan Normal University E-mail:tjkuo1215@gmail.com

#### Abstract

In this talk, I will first introduce a second order linear complex ODE so called the generalized Lamé equation (GLE) and discuss its monodromy representation. Secondly, I will focus on a class of mean field equation (MFE) which can induce a generalized Lamé equation. By applying the theory we develop in GLE, we could give a criterion of the existence of solutions to the MFE.

### On the uniqueness of the steady state in a Keller-Segel model

#### Wen Yang

#### Department of Applied Mathematics, Hong Kong Polytechnic University E-mail:math.yangwen@gmail.com

#### Abstract

We establish an inequality on the classical solutions of

$$\begin{cases} \Delta u - g(u) + f(u) = 0 & \text{ in } \Omega, \\ \partial_{\nu} u = 0 & \text{ on } \partial\Omega, \end{cases}$$

where  $f, f' > 0, g, g' \ge 0$ , which arises from the stationary Keller-Segel model in chemotaxis. As an application, we prove several uniqueness results for the Neumann boundary problem and provide the associated consequences for the time asymptotic behavior of the solutions to the corresponding time dependent chemotaxis systems. Furthermore, we study the related Dirichlet problem and obtain the optimal uniqueness result.

## Localized patterns and waves in FitzHugh-Nagumo type systems

#### Chao-Nien Chen

#### National Tsing Hua University E-mail:**chen@math.nthu.edu.tw**

#### Abstract

Localized patterns and waves are commonly observed in reaction-diffusion systems. Depending on the system parameters and initial conditions, such dissipative structures may stay at rest or propagate with a dynamically stabilized velocity. In this talk we aim at patterns and waves found in FitzHugh-Nagumo models.

## Wave interaction for reaction-diffusion equations with triple-well potential

#### Chiun-Chuan Chen and Chih-Chiang Huang

#### National Center for Theoretical Sciences, Mathematics Division E-mail:loveworldsteven@hotmail.com

#### Abstract

In this talk we will introduce a variational approach to construct traveling waves for reaction-diffusion equations. We study a classical combustion model which has a triple-well potential with three stable constant equilibrium 0, 1 and 2. In some situations, a traveling wave connecting 0 to 2(called 0-2 wave) can be viewed as an interaction between 0-1 wave and 1-2 wave. We will give a criterion to determine whether 0-2 wave exists. Moreover, a free boundary problem with this phenomenon is also studied.

## Ground states of spin-1 Bose-Einstein condensates, phase transitions and symmetry breaking

#### I-Liang Chern, Chiu-Fen Chou, Tien-Tsan Shieh

#### National Taiwan University E-mail:**tshieh@math.ntu.edu.tw**

#### Abstract

We develop an analytic theory for the ground state patterns and their phase transitions for spin-1 Bose-Einstein condensates on a bounded domain in the presence of a uniform magnetic field. Within the Thomas-Fermi approximation, these ground state patterns are composed of four basic states: magnetic state, nematic state, two-component state and three-component state, separated by interfaces. A complete phase diagram of the ground state patterns are found analytically with different quadratic Zeeman energy q and total magnetization M for both ferromagnetic and antiferromagnetic systems. Using the  $\Gamma$ -convergence technique, it is found that the semi-classical limits of these ground states minimize an energy functional which consists of interior interface energy plus a boundary contact energy. As a consequence, the interface between two different basic states has constant mean curvature, and the contact angle between the interface and the boundary obeys Young's relation.

## Effect of singular points on some parabolic evolutions and elliptic equations

#### Jann-Long Chern

#### National Central University, Taiwan E-mail:chern@math.ncu.edu.tw

#### Abstract

In this talk we will study the problem how the singular points affect the existence and structure of solutions for some parabolic evolutions and elliptic equations.

## Shock-wave structure for a polyatomic gas with large bulk viscosity based on kinetic theory

#### Kazuo Aoki<sup>a</sup> and Shingo Kosuge<sup>b</sup>

#### <sup>a</sup>NCTS, NTU and Department of Mathematics, NCKU E-mail:kazuo.aoki.22v@st.kyoto-u.ac.jp

#### <sup>b</sup>Center for Global Leadership Engineering Education, Kyoto University E-mail:kosuge.shingo.6r@kyoto-u.ac.jp

#### Abstract

The structure of a standing plane shock wave in a polyatomic gas is investigated on the basis of kinetic theory, with special interest in gases with large bulk viscosities, such as the  $CO_2$  gas. The ellipsoidal statistical (ES) model for a polyatomic gas is employed. First, the shock structure is computed numerically for various upstream Mach numbers and for various (large) values of the ratio of the bulk viscosity to the shear viscosity, and different types of profiles, such as the double-layer structure consisting of a thin upstream layer with a steep change and a much thicker downstream layer with a mild change, are obtained. Then, an asymptotic analysis for large values of the ratio is carried out for the ES model, and a set of macroscopic equations, the solution of which describes the different types of profiles obtained by the numerical analysis correctly, is derived.

## Regularity for diffuse reflection boundary problem to the stationary linearized Boltzmann equation in a convex domain

I-Kun Chen\*, Chun-Hsiung Hsia, Daisuke Kawagoe

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#### Abstract

We consider the diffuse reflection boundary problem for linearized Boltzmann equation for hard sphere potential, cutoff hard potential, or Maxwellian molecular gases in a  $C^2$  strictly convex bounded domain. We obtain a pointwise estimate for the derivative of the solution provided the boundary temperature is bounded differentiable and the solution is bounded. Velocity averaging effect for stationary solutions as well as observations in geometry are used in this research.

## On the global dynamics of the Boltzmann equation

#### Jin-Cheng Jiang

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#### Abstract

We will represent a new result on the global dynamics of the Boltzmann equation for non-cutoff model based on the joint work with Lingbing He.

Use the energy-entropy method, we are able to characterizes the propagation of the regularity, H-theorem and the interplay between the energy and the entropy. Our main results including the followings.

• We present a unified framework to prove the well-posedness for the original Boltzmann equation for both angular cutoff and without cutoff in weighted Sobolev spaces with polynomial weights. As a consequence, we obtain an explicit formula for the asymptotics of the equation from angular cutoff to non-cutoff.

• We describe the global dynamics of the equation under the almost optimal assumption on the solution which makes sure that the Boltzmann collision operator behaves like a fractional Laplace operator for the velocity variable. More precisely, we obtain the propagation of the full regularity or the smoothing effect for the solution and a new mechanism for the convergence of the solution to the equilibrium with quantitative estimates.

• We prove that any global and smooth solution to the equation is stable, i.e., any perturbed solution will keep close to the reference solution if initially they are close to each other.

## Effect of abrupt change of the wall temperature in the kinetic theory

#### Hung-Wen Kuo

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#### Abstract

We investigate the response of a dilute gas to the abrupt change of the temperature of the bounding wall on the basis of the Boltzmann equation. Consider a semi-infinite expanse of a rarefied gas with density  $\rho_0$  bounded by an infinite plane wall. The gas is initially in equilibrium with the bounding gas at temperature  $T_0$ . We study the asymptotic behavior of the gas when the temperature of the wall is suddenly changed to  $T_w$  at time t = 0 and then is kept constant. We show that for short times the solution represents a perturbation to the linearized free molecular flow. We also obtain the asymptotic expansion of the solution for large times.

## Global well-posedness of the anisotropic primitive equations

#### Jinkai Li

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#### Abstract

The motion of the large-scale atmospheric and oceanic flows is governed by the primitive equations (PEs), which are derived from the Navier-Stokes equations by using the Boussinesq and hydrostatic approximations. The strong horizontal turbulent mixing, which creates the horizontal eddy viscosity, leads us to consider the PEs with horizontal viscosity. It will be shown that the 3D PEs with horizontal viscosity admits a unique global strong solution, for arbitrary sufficient smooth initial data, as long as one still has the horizontal or vertical thermal diffusivity. These are joint works with Chongsheng Cao and Edriss S. Titi.

### Ground states to relativistic quantum mean field equations

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#### Abstract

Through a mean field limit, the dynamics of a system consisting of a huge number bosons is approximately described by the following mean field equation

$$i\partial_t \psi = -\frac{1}{2m} \Delta \psi - (V * |\psi|^2)\psi, \qquad (1)$$

where V is a two-body interaction potential. If we choose V as Newtonian potential, we obtain the nonlinear Hartree equation and if V is chosen as the Dirac-delta distribution, the nonlinear Schrödinger equation is derived. The ground state to (1) is referred as a stationary solution of (1) which represents a quantum state carrying the least possible energy. During several decades, a great deal of work has been devoted to the study of existence and qualitative properties of ground states to (1).

For relativistic bosons, there is one of the relativistic counterparts to (1), known as the pseudo-relativisitic quantum mean field equation:

$$i\partial_t \psi = \sqrt{-c^2 \Delta + m^2 c^4} \psi - (V * |\psi|^2) \psi, \qquad (2)$$

Especially, the equation (2) is used to model a hypothetical astronomical object, a boson star.

In this talk, I will review the existence and qualitative properties of ground states to (1) and discuss about what happens when we turn to (2). We are also particularly interested in the non-relativistic limit, which takes the speed of light c to infinite so that non-relativistic physics is recovered. We shall explicitly calculate the order and rate of convergence of the non-relativistic limit between ground states to the pseudo-relativistic and non-relativistic mean field equations.

### Stokes expansions and asymptotic models of water waves

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#### Abstract

The study of irrotational incompressible Euler equations has been a long tradition in the fluid community. When the free surface was taken into account, both the theoretical study and robust numerical schemes become very challenging, especially for the case of deep water (Euler equations on a fluid domain with infinite depth). In the numerical side, various methods used to compute the Dirichlet-to-Neumann map (which is highly related to the water wave equations) proposed by W. Craig et al (1993) and M.J. Ablowitz et al (2006, 2008) involve highly illconditioned intermediate calculations (while the difficulties can be overcome by implementing multiple-precision arithmetic). The boundary integral collocation method and the transformed field expansion method are then introduced to avoid catastrophic cancellation of digits in the intermediate results; however, carrying out those methods in the three-dimensional case seems difficult. Therefore, the search for good asymptotic models for water waves become appealing for it might provide models that can be easily implemented and at the same time provide accurate enough evolution of the free surface. B. Akers et al (2010) propose a quadratic approximation of the water wave equation; however, the derivation of such a model seems not rigorous. In this talk, I will present how the Stokes expansions can be used to derive asymptotic models up to any order.

## Global well-posedness of Cauchy problem for compressible Euler equations in transonic nozzle flow

#### Meng Kai (John) Hong

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#### Abstract

In this talk, we consider the Cauchy problem of one-dimensional compressible Euler system for the transonic nozzle flow. We provide the global well-posedness of such problem for the case of expanding nozzles. The global existence of entropy solution is established by the generalized Glimm method. The stability of solution is obtained by extending the results of Bressan, Ha, Liu and Yang to the case of which subsonic and supersonic states both exist.

## Forest population dynamics models

#### Pierre Magal

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#### Abstract

In the first part of this presentation we will present a mathematical model for forest growth and we compare this model with a computer forest simulator named SORTIE. The main ingredient taken into account in both models is the competition for light between trees. The parameters of the mathematical model are estimated by using SORTIE model, when the parameter values of SORTIE model correspond to the ones previously evaluated for the Great Mountain Forest in USA. We will see that the best fit of the parameters of the mathematical model is obtained when the competition for light influences only the growth rate of trees. We will construct a size structured population dynamics model with one and two species and with spatial structure.

The second part of the talk a pine tree forest with a parasite called nematode. Since this parasite colonizes pine trees to reproduce, it is natural to introduce a predator-prey (or consumer-resource) relationship between the trees and the parasites. In order to investigate the behaviour of the resulting system, we will use numerical simulations, and we will introduce a parasite into a population of trees that: 1) is not oscillating around the positive equilibrium; 2) has some damped oscillations; 3) has some undamped oscillations. This will correspond to three scenarios for parameter values. As one may expect, this will lead to complex dynamics, since we combine the oscillations produced by the predator-prey system with the oscillations coming from the demographic properties of the prey.

In the last part of the presentation, we will consider a class state-dependent delay differential equations with infinite delay. This class will covers the above examples of models for forest. We will discuss the existence and uniqueness of a maximal semiflow in a weighted space of both Lipschitz functions and  $C^1$  functions. We will obtain a blow-up result when the time approaches the maximal time of existence. We will conclude the presentation with an application to prove the existence of global positive solutions for a spatially structured forest model.

## Concentration phenomena and curvature effects on the structure of thin electrical double layers

#### Chiun-Chang Lee

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#### Abstract

For the structure of the thin electrical double layer (EDL) and the property related to the EDL capacitance, we analyze boundary layer solutions (for the electrostatic potential) of a charge-conserving Poisson–Boltzmann (CCPB) equation which is a steady-state Poisson– Nernst–Planck equation with a singular perturbation parameter related to the small Debye screening length. Theoretically, the boundary layer solutions describe that those ions exactly approach neutrality in the bulk, and the extra charges are accumulated near the charged surface. Hence, the non-neutral phenomenon merely occurs near the charged surface. In this talk, we introduce new analysis techniques to investigate thin boundary layer structures. A series of fine estimates combining the Pohozaev's identity, the inverse Hölder type estimates and some technical comparison arguments are developed in arbitrary bounded domains. We further concentrate on the physical domain being a ball with the simplest geometry and gain a clear picture on the effect of the curvature on the boundary layer solutions. As an application, we provide a theoretical way to support that the EDL has higher capacitance in a quite thin region near the charged surface, not in the whole EDL. In particular, for the cylindrical electrode, our result has a same analogous measurement as the specific capacitance of the well-known Helmholtz double layer.

## Structural and dynamical analysis of social networks using isospectral reductions

#### Chi-Jen Wang, Seokjoo Chae, Leonid A. Bunimovich, Benjamin Z. Webb

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#### Abstract

We employ the recently developed theory of isospectral network reductions to analyze multi-mode social networks. This procedure allows us to uncover the hierarchical structure of the networks we consider as well as the hierarchical structure of each mode of the network. Additionally, by performing a dynamical analysis of these networks we are able to analyze the evolution of their structure allowing us to find a number of other network features. We apply both of these approaches to the Southern Women Data Set, one of the most studied social networks and demonstrate that these techniques provide new information, which complements previous findings.

## Efficient and flexible implementation of Langevin simulation for gene burst production

## Ching-Cher Sanders Yan, Surendhar Reddy Chepyala, Chao-Ming Yen, Chao-Ping Hsu

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#### Abstract

Gene expression involves production of mRNA and protein both in bursts, and the fluctuations in their number are increased due to such bursts. The Langevin equation is an efficient and versatile means to simulate such number fluctuation. However, how to include these mRNA and protein bursts in the Langevin equation is not intuitively clear. In this work, we estimated the variance in burst production from a two-state gene expression model and introduced such variation in the Langevin equation. Our approach offers different Langevin expressions for either or both transcriptional and translational bursts considered and saves computer time by including many production events at once in a short time step. The errors can be controlled to be rather precise (< 2%) for the mean and < 10% for the standard deviation of the steady-state distribution. Our scheme allows for high-quality stochastic simulations with the Langevin equation for gene expression, which is useful in analysis of biological networks.

## Variational nature of Keplerian orbits

#### Kuo-Chang Chen

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#### Abstract

Keplerian orbits can be characterized as minimizers of some action functional on function spaces with natural topological or boundary constraints. This fact is useful in variational construction of periodic orbits for the n-body and n-center problems. The elliptic case, settled by W. Gordon in 1977, is considerably well-known. Parabolic case is less well-known, and hyperbolic case is virtually unknown. In this talk I will briefly outline those known facts, and describe my proof for the minimizing property of hyperbolic orbits.

### Structure-preserving flows of symplectic matrix pairs

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#### Abstract

We construct a nonlinear differential equation of matrix pairs  $(\mathcal{M}(t), \mathcal{L}(t))$  that is invariant (the **Structure-Preserving Property**) in the class of symplectic matrix pairs

$$\mathbb{S}_{\mathcal{S}_1,\mathcal{S}_2} = \left\{ (\mathcal{M},\mathcal{L}) \mid \mathcal{M} = \begin{bmatrix} X_{12} & 0 \\ X_{22} & I \end{bmatrix} \mathcal{S}_2, \mathcal{L} = \begin{bmatrix} I & X_{11} \\ 0 & X_{21} \end{bmatrix} \mathcal{S}_1 \text{ and } X = [X_{ij}]_{1 \le i,j \le 2} \in \mathbb{H}(2n) \right\}$$

for certain fixed symplectic matrices  $S_1$  and  $S_2$ . Its solution also preserves invariant subspaces on the whole orbit (the **Eigenvector-Preserving Property**). Such a flow is called a *structure-preserving flow* and is governed by a Riccati differential equation (RDE) having the form

$$\dot{W}(t) = [-W(t), I] \mathscr{H}[I, W(t)^{\top}]^{\top}, W(0) = W_0,$$

for some suitable Hamiltonian matrix  $\mathscr{H}$ . In addition, Radon's lemma (see Theorem ??) leads to the explicit form  $W(t) = P(t)Q(t)^{-1}$  where  $[Q(t)^{\top}, P(t)^{\top}]^{\top} = e^{\mathscr{H}t}[I, W_0^{\top}]^{\top}$ . Therefore, blow-ups for the structure-preserving flows may happen at a finite t whenever Q(t) is singular. To continue, we then utilize the Grassmann manifolds to extend the domain of the structurepreserving flow to the whole  $\mathbb{R}$  subtracting some isolated points. On the other hand, the Structure-Preserving Doubling Algorithm (SDA) is an efficient numerical method for solving algebraic Riccati equations and nonlinear matrix equations. In conjunction with the structurepreserving flow, we consider the following two special classes of symplectic pairs:  $\mathbb{S}_1 = \mathbb{S}_{I_{2n},I_{2n}}$ and  $\mathbb{S}_2 = \mathbb{S}_{\mathcal{J},-I_{2n}}$  and the corresponding algorithms SDA-1 and SDA-2. It is shown that at  $t = 2^{k-1}, k \in \mathbb{Z}$  this flow passes through the iterates generated by SDA-1 and SDA-2, respectively. Therefore, the SDA and its corresponding structure-preserving flow have identical asymptotic behaviors, including the stability, instability, periodicity, and quasi-periodicity of the dynamics.

## A note on holomorphic shadowing for Hénon maps

#### Yi-Chiuan Chen

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#### Abstract

In studying the complex Hénon maps, Mummert defined an operator the fixed points of which give rise to bounded orbits. This enabled him to obtain an estimate of the solenoid locus. Instead of the contraction mapping theorem, in the talk, I shall present an implicit function theorem version of his result, with some generalisation.

## Relative equilibria of the planar four-vortex problem

#### Ya-Lun Tsai

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#### Abstract

The motion of point vortices in the plane is an old problem of fluid mechanics. Relative equilibria are rigidly rotating configurations. In this talk, I will focus on enumerating the numbers of such solutions for some four-vortex problems involving two vorticities parameters.

## Quantitative estimate for the Lamé system with rough coefficients

#### C.L. Lin, H. Koch and J.N. Wang

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#### Abstract

In this talk we study the local behavior of a solution to the Lamé system when the Lamé coefficients  $\lambda$  and  $\mu$  satisfy that  $\mu$  is Lipschitz and  $\lambda$  is essentially bounded in dimension  $n \geq 2$ . One of the main results is the *local* doubling inequality for the solution of the Lamé system. This is a quantitative estimate of the strong unique continuation property. Our proof relies on Carleman estimates with carefully chosen weights.

## Inverse transmission scattering problem via a Dirichlet-to-Neumann map

#### Kuo-Ming Lee

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#### Abstract

In this talk, we consider the inverse scattering problem of a time-harmonic wave from a penetrable obstacle. The aim is to recover the shape and the location of the scatterer from the far-field data. Usually, the solving of the scattering problem involves an interior and an exterior boundary value problem. From the viewpoint of the inverse problem, this is not particular practical.

We develop a Dirichlet-to-Neumann map to convert this problem into an exterior boundary value problem with an impedance-like boundary condition. The problem is then solved without the need of solving of an interior problem. This process reduces the size of the problem and thus enables an efficient treatment of this inverse scattering problem.

## Strong unique continuation for two-dimensional anisotropic elliptic systems

#### Rulin Kuan

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#### Abstract

In this work, we give the strong unique continuation property for a general two dimensional anisotropic elliptic system with real coefficients in a Gevrey class under the assumption that the principal symbol of the system has simple characteristics. The strong unique continuation property is derived by obtaining some Carleman estimate. The derivation of the Carleman estimate is based on transforming the system to a larger second order elliptic system with diagonal principal part which has complex coefficients.

### The nonlocal Calderón problem

#### Yi-Hsuan Lin

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#### Abstract

We review recent progress in the fractional Calderón problem, where one tries to determine an unknown coefficient in a nonlocal Schrödinger type equation from exterior measurements of solutions. This equation enjoys remarkable uniqueness and approximation properties, which turn out to yield strong results in related inverse problems.



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